



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 11

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2019

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 21 pages./
Hierdie nasienriglyne bestaan uit 21 bladsye.**

NOTE:

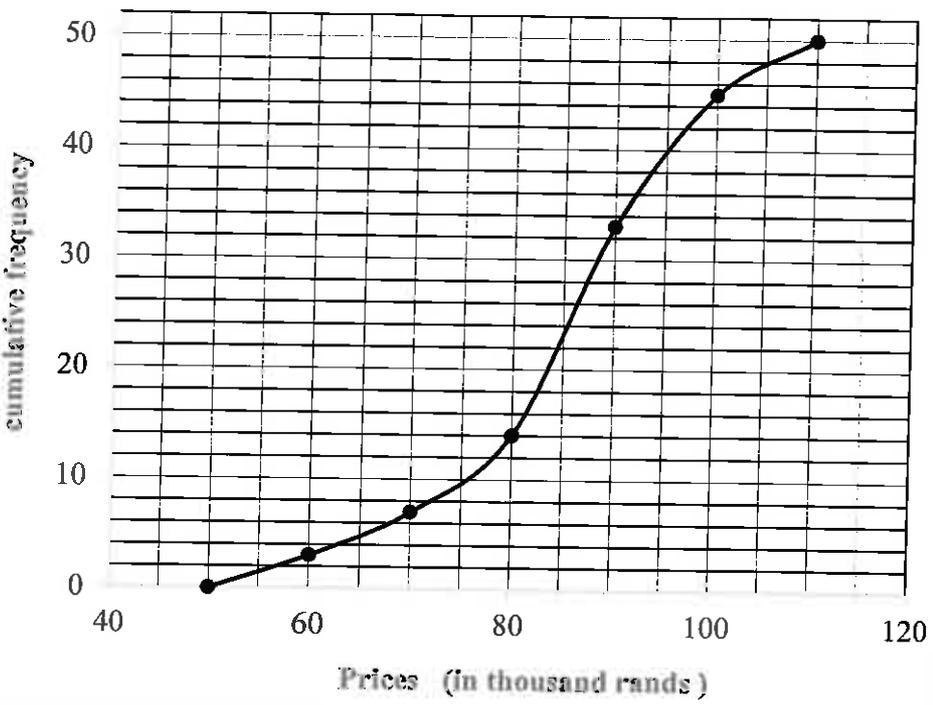
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

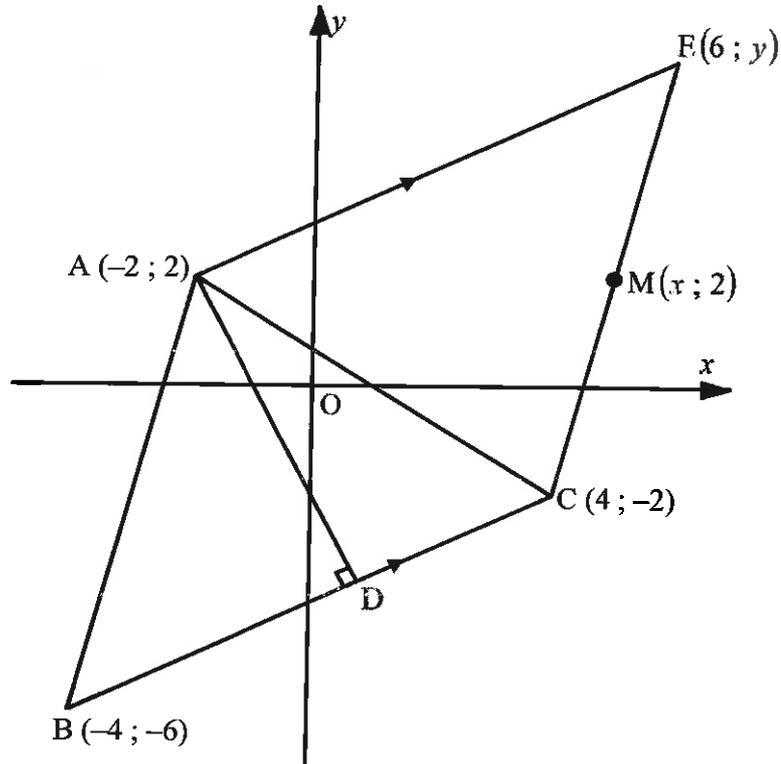
- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

QUESTION/VRAAG 1

1.1	SELLING PRICE/ VERKOOPSPRYS (IN THOUSANDS OF RANDS/ IN DUISENDE RAND)	FREQUENCY/ FREKWENSIE	CUMULATIVE FREQUENCY/ KUMULATIEWE FREKWENSIE	
	$50 \leq x < 60$	3	3	
	$60 \leq x < 70$	4	7	$\checkmark a = 7$
	$70 \leq x < 80$	7	14	
	$80 \leq x < 90$	19	33	
	$90 \leq x < 100$	12	45	$\checkmark b = 45$
	$100 \leq x < 110$	5	50	
				(2)

<p>1.2</p>	<p style="text-align: center;">Cumulative frequency graph on the price of cars</p> 	<ul style="list-style-type: none"> ✓ grounding/ geanker (50:0) ✓ upper boundry/ boonste grens ✓ smooth curve/ gladde kurwe <p style="text-align: right;">(3)</p>
<p>1.3</p>	<p>(95 000 ; 40) 40 cars to choose from/40 motors om vanuit te kies (Accept any answer between 37 and 43/ Aanvaar enige antwoord tussen 37 en 43)</p>	<p style="text-align: right;">(1)</p>
		<p style="text-align: right;">[6]</p>

QUESTION/VRAAG 3

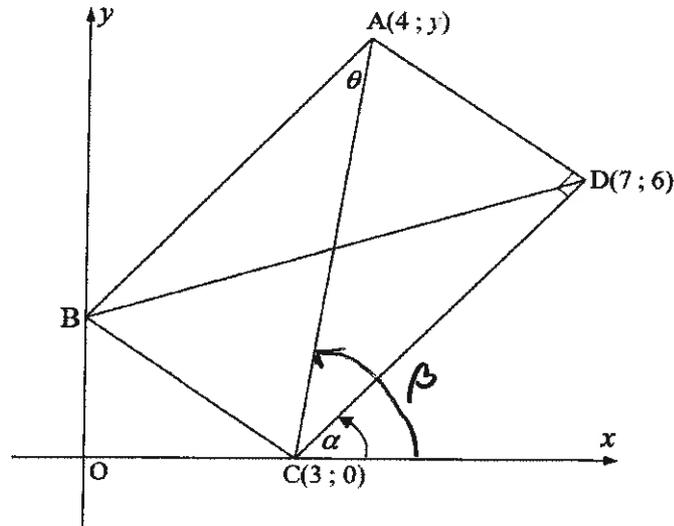


<p>3.1</p>	<p>B(-4;-6) C(4;-2)</p> $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2 - (-6)}{4 - (-4)}$ $= \frac{4}{8}$ $= \frac{1}{2}$ <p style="text-align: center;">OR OR</p> $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-6 - (-2)}{-4 - 4}$ $= \frac{-4}{-8}$ $= \frac{1}{2}$	<p>✓ gradient formula/ gradiëntformule</p> <p>✓ subst into/ vervanging in gradient form./ gradiëntform.</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(3)</p>
<p>3.2</p>	$x = \frac{6+4}{2}$ $x = 5$ $\frac{y+(-2)}{2} = 2$ $y = 6$	<p>✓ $x = 5$</p> <p>✓ $\frac{y+(-2)}{2} = 2$</p> <p>✓ $y = 6$</p> <p style="text-align: right;">(3)</p>

3.3	$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{((-4) - 4)^2 + ((-6) - (-2))^2}$ $= \sqrt{80}$	✓ subst. in distance formula/verv. in afstandformule ✓ answer/antwoord (2)
3.4	Parallelogram	✓ answer/antwoord (1)
3.5	$m_{AD} = -2 \text{ (AD} \perp \text{BC)}$ $y = -2x + c$ $2 = -2(-2) + c \quad \text{sub } A(-2; 2)$ $-2 = c$ $\therefore y = -2x - 2$ <p>OR/OF</p> $m_{AD} = -2 \text{ (AD} \perp \text{BC)}$ $y - 2 = -2(x - (-2))$ $y - 2 = -2x - 4$ $\therefore y = -2x - 2$	✓ $m_{AD} = -2$ ✓ subst. of/verv. m and point/en punt $(-2; 2)$ / ✓ answer/antwoord ✓ $m_{AD} = -2$ ✓ subst. of/verv. m and point/en punt $(-2; 2)$ ✓ answer/antwoord (3)

<p>3.6</p>	$m_{BC} = \frac{1}{2}$ <p>Equation of/Verg. van BC:</p> $y = \frac{1}{2}x + c$ $-2 = \frac{1}{2}(4) + c$ $-4 = c$ $\therefore y = \frac{1}{2}x - 4$ $-2x - 2 = \frac{1}{2}x - 4$ $-4x - 4 = x - 8$ $4 = 5x$ $x = \frac{4}{5}$ $y = (-2)\frac{4}{5} - 2$ $= \frac{-18}{5}$ <p>D($\frac{4}{5}; \frac{-18}{5}$)</p> $y - (-2) = \frac{1}{2}(x - 4)$ $y + 2 = \frac{1}{2}x - 2$ $\therefore y = \frac{1}{2}x - 4$ $y = \frac{1}{2}\left(\frac{4}{5}\right) - 4$ $= \frac{-18}{5}$ <p style="text-align: center;">OR/OF</p>	<p>✓ subst. of/verv. m and point/en punt (4;-2)/</p> <p>✓ $y = \frac{1}{2}x - 4$</p> <p>✓ equating both equations/vergelyking van beide vergelykings</p> <p>✓ x-value/-waarde</p> <p>✓ y-value/-waarde</p> <p style="text-align: right;">(5)</p>
<p>3.7</p>	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\left(\frac{4}{5}\right) - (-2)\right)^2 + \left(\left(\frac{-18}{5}\right) - 2\right)^2}$ $= \frac{14\sqrt{5}}{5}$ <p>Area of/Oppervlakte van $\Delta AEC = \frac{1}{2}AE \times AD$</p> $= \frac{1}{2} \times \sqrt{80} \times \frac{14\sqrt{5}}{5}$ $= 28 \text{ units}^2 / \text{eenhede}^2$	<p>✓ length of AD/ lengte van AD</p> <p>✓ subst into area formula</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(3)</p>
[20]		

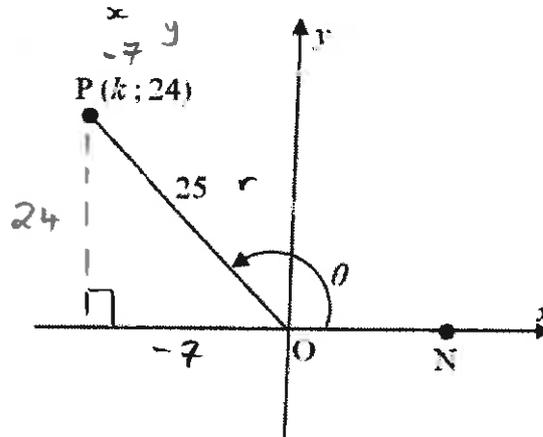
QUESTION/VRAAG 4



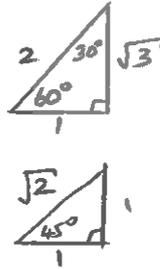
4.1	$m_{DC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{6 - 0}{7 - 3}$ $= \frac{3}{2}$	✓ substitution in gradient formula/ vervanging in gradiëntformule ✓ $m_{DC} = \frac{3}{2}$ (2)
4.2	$\tan \alpha = m_{CD}$ $\tan \alpha = \frac{3}{2}$ I : $\alpha = 56,31^\circ$ <i>mf = 56,30...° tan + m</i>	✓ $\tan \alpha = \frac{3}{2}$ ✓ $\alpha = 56,31^\circ$ (2)
4.3	$m_{CD} \times m_{AD} = -1$ $\frac{3}{2} \times \frac{y-6}{4-7} = -1$ $\frac{y-6}{-3} = -\frac{2}{3}$ $3y - 18 = 6$ $y - 6 = 2$ $y = 8$	✓ $\frac{y-6}{-3} = -\frac{2}{3}$ ✓ $3y - 18 = 6$ ✓ $y = 8$ (4)

4.4	$m_{AC} = \frac{8-0}{4-3} = 8$ $\therefore \text{Inclination of AC} = \tan^{-1}(8)$ $\hat{\angle} : \beta = 82,87^\circ$ $\hat{A}CD = 82,87^\circ - 56,31^\circ$ $= 26,56^\circ$ $\therefore \theta = 26,56^\circ \quad \text{alt}'s =, AB \parallel CD$	$\checkmark m_{AC} = 8$ $\checkmark 82,87^\circ$ $\checkmark \hat{A}CD = 82,87^\circ - 56,31^\circ$ $\checkmark \hat{A}CD = 26,56^\circ$ $\checkmark \theta = 26,56^\circ$
		(5)
		[13]

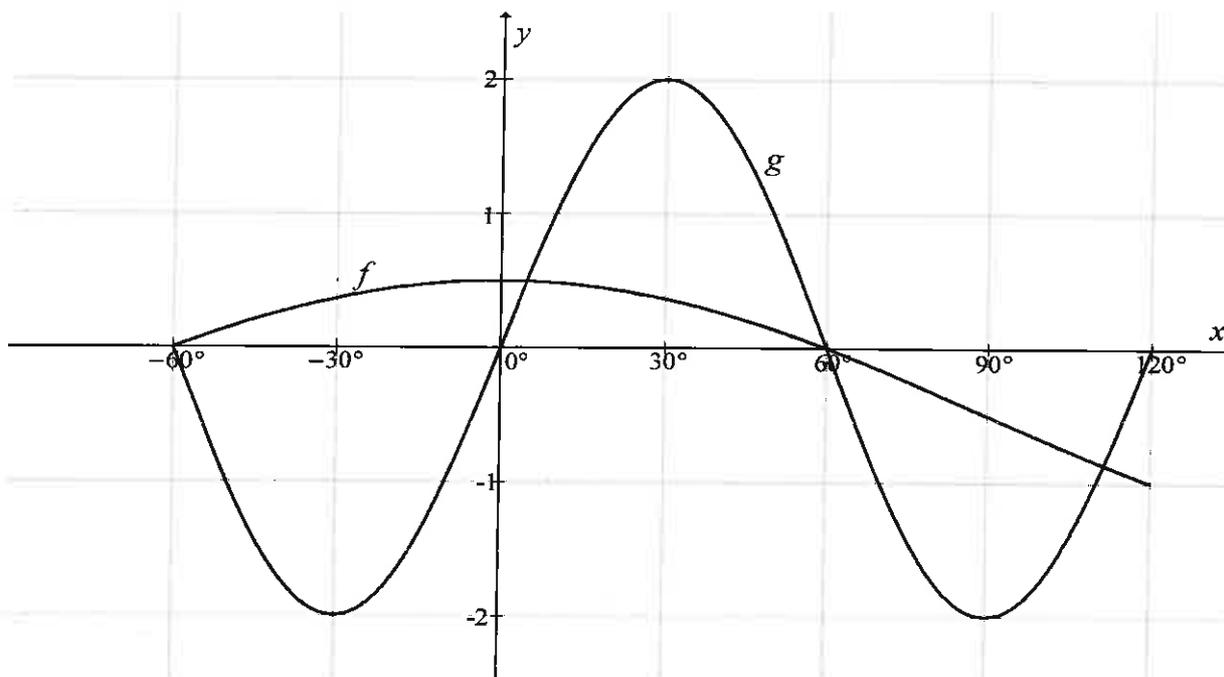
QUESTION/VRAAG 5



5.1.1	$x^2 + y^2 = r^2$ $(k)^2 + (24)^2 = 25^2$ $k^2 = 49$ $k = -7$ <p style="text-align: right;"><i>Pythag</i></p>	✓ subst./ vervanging ✓ answer/antwoord (2)
5.1.2	$\tan \theta = -\frac{24}{7}$ <p style="text-align: right;"><i>x/y</i></p>	✓ answer/antwoord (1)
5.1.3	$\theta + \alpha = 360^\circ$ $\alpha = 360^\circ - \theta$ $\sin \alpha = \sin(360^\circ - \theta)$ $= -\sin \theta$ $= -\frac{24}{25}$ <p style="text-align: right;"><i>1/2</i></p>	✓ $\sin \alpha = \sin(360^\circ - \theta)$ ✓ $-\sin \theta$ ✓ answer/antwoord (3)
5.1.4	$\cos^2 \theta - \sin^2 \alpha$ $= \left(\frac{-7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2$ $= \frac{-527}{625}$ <p style="text-align: right;">$\left(\frac{x}{r}\right)^2$ $\left(\frac{y}{r}\right)^2$</p>	✓✓ substitution/ vervanging ✓ answer/antwoord (3)

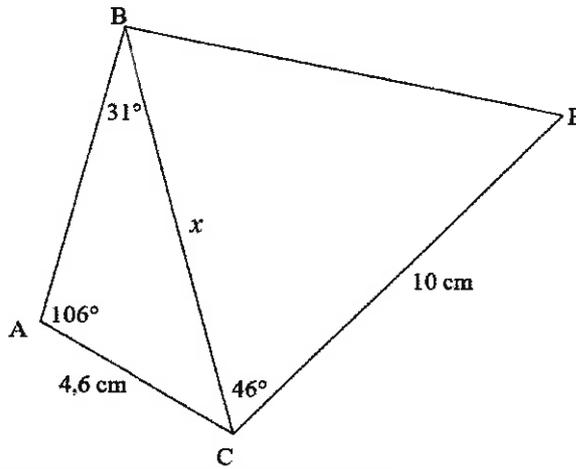
<p>5.2</p>	$\frac{\cos 210^\circ \cdot \tan 135^\circ}{\sin(-60^\circ) \cdot \cos 420^\circ}$ $= \frac{-\cos 30^\circ \cdot -\tan 45^\circ}{-\sin 60^\circ \cdot \cos 60^\circ}$ $= \frac{\left(-\frac{\sqrt{3}}{2}\right)(-1)}{\left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2}}$ $= -2$  <p>no Δ or incorrect Δ, no marks max 2/5</p>	<p>✓ $-\cos 30^\circ \cdot -\tan 45^\circ$ ✓ $-\sin 60^\circ \cdot \cos 60^\circ$ ✓ $\left(-\frac{\sqrt{3}}{2}\right)(-1)$ ✓ $\left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2}$ ✓ answer/antwoord (5)</p>
<p>5.3</p>	$\text{LHS} = \frac{1}{\tan^2 x} - \cos^2 x$ $= \frac{1}{\frac{\sin^2 x}{\cos^2 x}} - \cos^2 x$ $= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$ $= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$ $= \frac{\cos^2 x(1 - \sin^2 x)}{\sin^2 x}$ $= \frac{\cos^2 x(\cos^2 x)}{\sin^2 x}$ $= \frac{\cos^4 x}{\sin^2 x}$ <p>= RHS</p>	<p>✓ $\frac{\sin^2 x}{\cos^2 x}$ ✓ common denominator/ gemene noemer ✓ factors/faktore ✓ $1 - \sin^2 x = \cos^2 x$ (4)</p>
<p>5.4</p>	$\sqrt{2} \sin x \cos x = \cos x$ $\sqrt{2} \sin x \cos x - \cos x = 0$ $\cos x(\sqrt{2} \sin x - 1) = 0$ <p>$\cos x = 0$ $x = 90^\circ + 360^\circ k, k \in \mathbb{Z}$ or $x = 270^\circ + 360^\circ k, k \in \mathbb{Z}$</p> <p>OR $x = 90^\circ + 180^\circ k, k \in \mathbb{Z}$</p> <p>both $\sin x = \frac{1}{\sqrt{2}}$ ref $\angle = 45^\circ$ sin + in I: $x = 45^\circ + 360^\circ k, k \in \mathbb{Z}$ or II: $x = 135^\circ + 360^\circ k, k \in \mathbb{Z}$</p> <p>no $k \in \mathbb{Z}, -1$</p>	<p>✓ standard form/stand.vorm ✓ factors/faktore ✓ both equations/beide vergelykings ✓ $x = 90^\circ + 360^\circ k, k \in \mathbb{Z}$ ✓ $x = 45^\circ + 360^\circ k, k \in \mathbb{Z}$ ✓ $x = 135^\circ + 360^\circ k, k \in \mathbb{Z}$ (6) [24]</p>

QUESTION/VRAAG 6

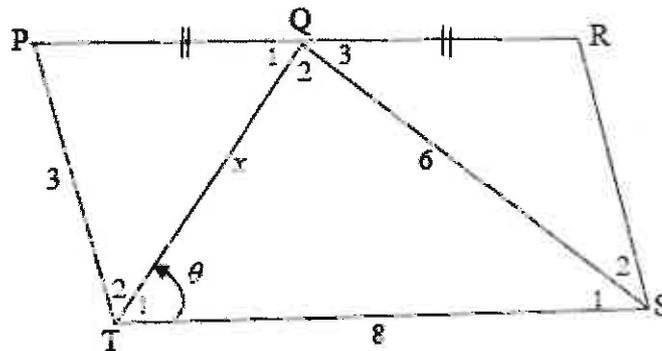


6.1	(30°:2)	✓ answer/antwoord (1)
6.2	$-60^\circ < x < 0^\circ$ $x \in (-60^\circ; 0^\circ)$ int ✓ not ✓ • only award not mark if int correct	✓ endpoints/eindpunte ✓ notation/notasie (2)
6.3	$q = -\frac{1}{2}$ $b = 3$	✓ $q = -\frac{1}{2}$ ✓ $b = 3$ (2)
6.4	$2 \cos x \sin 3x - \sin 3x \geq 0$ $2 \sin 3x \left(\cos x - \frac{1}{2} \right) \geq 0$ $g(x).f(x) \geq 0$ $0^\circ \leq x \leq 120^\circ$ or $x = -60^\circ$	✓✓ $2 \sin 3x \left(\cos x - \frac{1}{2} \right) \geq 0$ ✓ ✓ (4)
		[9]

QUESTION/VRAAG 7



7.1.1	$\frac{BC}{\sin \hat{A}} = \frac{AC}{\sin \hat{B}}$ $\frac{x}{\sin 106^\circ} = \frac{4,6}{\sin 31^\circ}$ $x = \frac{4,6 \sin 106^\circ}{\sin 31^\circ}$ $x = 8,59 \text{ cm}$	✓ substitution into sine rule/vervang in sinusreël ✓ $x = \frac{4,6 \sin 106^\circ}{\sin 31^\circ}$ ✓ answer/antwoord (3)
7.1.2	$\hat{A}CB = 43^\circ$ <i>Sum \hat{A}s in $\Delta = 180^\circ$</i> $\text{Area ACEB} = \left(\frac{1}{2} \times AC \times BC \times \sin \hat{A}CB \right) + \left(\frac{1}{2} \times BC \times CE \times \sin \hat{B}CE \right)$ $= \left(\frac{1}{2} \times 4,6 \times 8,59 \times \sin 43^\circ \right) + \left(\frac{1}{2} \times 8,59 \times 10 \times \sin 46^\circ \right)$ $= 44,37 \text{ cm}^2$	✓ $\hat{A}CB = 43^\circ$ ✓✓ substitution/vervang ✓ answer/antwoord (4)

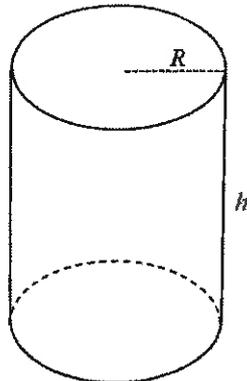


<p>7.2.1</p>	<p>In ΔQTS</p> $6^2 = 8^2 + x^2 - 2(8)(x)\cos\theta$ $16x\cos\theta = 8^2 + x^2 - 6^2$ $16x\cos\theta = x^2 + 28$ $\cos\theta = \frac{28 + x^2}{16x}$	<p>✓✓ substitution into cosine rule/ vervanging in cosinusreël ✓ $16x\cos\theta = x^2 + 28$</p> <p>(3)</p>
<p>7.2.2</p>	<p>$\hat{Q}_1 = \theta$ and $PQ = 4$</p> <p>In ΔQTP</p> $3^2 = 4^2 + x^2 - 2(4)(x)\cos\theta$ $8x\cos\theta = 4^2 + x^2 - 3^2$ $8x\cos\theta = x^2 + 7$ $\cos\theta = \frac{7 + x^2}{8x}$ $\frac{7 + x^2}{8x} = \frac{28 + x^2}{16x}$ $112x + 16x^3 = 224x + 8x^3$ $8x^3 = 112x$ $8x^3 - 112x = 0$ $8x(x^2 - 14) = 0$ $x \neq 0 \quad x = \sqrt{14}$	<p>✓</p> <p>$\hat{Q}_1 = \theta$ and $PQ = 4$</p> <p>✓</p> $3^2 = 4^2 + x^2 - 2(4)(x)\cos\theta$ $\checkmark \cos\theta = \frac{7 + x^2}{8x}$ <p>✓ equating/ vergelyking</p> <p>✓ factors/faktore</p> <p>✓ $x \neq 0 \quad x = \sqrt{14}$</p> <p>(6)</p>
		<p>[16]</p>

QUESTION/VRAAG 8

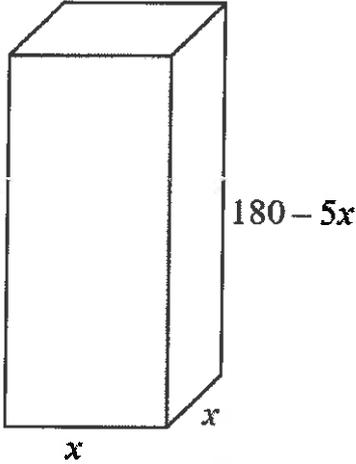


marble
alabaster



Volume of sphere = $\frac{4}{3}\pi r^3$
 Surface area of a sphere = $4\pi r^2$

Volume of sfeer = $\frac{4}{3}\pi r^3$
 Oppervlakte van 'n sfeer = $4\pi r^2$

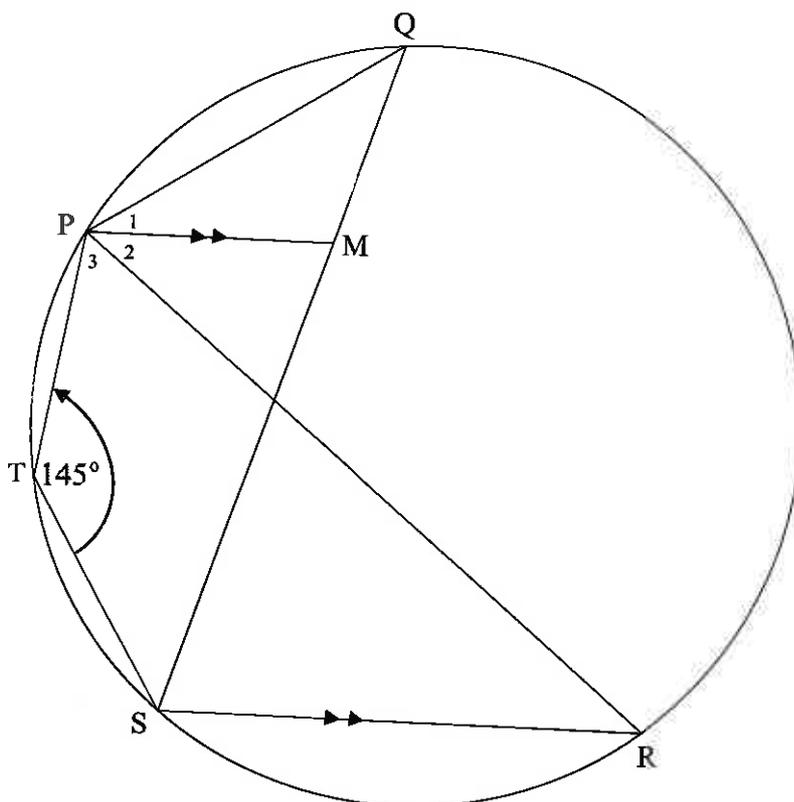
8.1.1	$V = \pi r^2 h$ $300 = \pi R^2 h$ $h = \frac{300}{\pi R^2}$	✓ substitution/ vervanging ✓ isolating h/ isoleer h (2)
8.1.2	$\text{Volume of marbles} = 100 \left(\frac{4}{3} \pi r^3 \right)$ $= 100 \times \frac{4}{3} \pi (0.75)^3$ $= \frac{225\pi}{4}$ $= 176,71 \text{ cm}^3$ <p>amount of water = $300 - 176,71$</p> $= 123,29 \text{ cm}^3$ 	✓ subst. into formula/vervanging in formule ✓ 176,71 ✓ 123,29 cm ³ (3)

8.2	<p>TSA = $2 \times$ area of the base + (perimeter \times height)</p> <p>TSA = $2 \times$ oppervlakte van basis \pm (omtrek \times hoogte)</p> <p>$= 2(x^2) + 4x(180 - 5x)$</p> <p>$= 2x^2 + 720x - 20x^2$</p> <p>$= -18x^2 + 720x$</p> <p>Surface area will be maximum when</p> <p>Oppervlakte sal op maksimum wees wanneer</p> <p>$x = \frac{-b}{2a} = \frac{-720}{2(-18)} = 20$</p> <p>max. surface area/maks. oppervlakte = $-18(20)^2 + 720(20)$</p> <p style="text-align: center;">$= 7200 \text{ cm}^2$</p>	<p>✓ subst. into formula/vervanging in formule</p> <p>✓ simplification/vereenvoudiging</p> <p>✓ value of/waarde van x</p> <p>✓ subst./verv $x = 20$</p> <p>✓ answer/antwoord (5)</p> <p style="text-align: right;">[10]</p>
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$$TSA = 2x \begin{array}{|c|} \hline x \\ \hline \end{array} x + 4x \begin{array}{|c|} \hline 180 - x \\ \hline x \\ \hline \end{array}$$

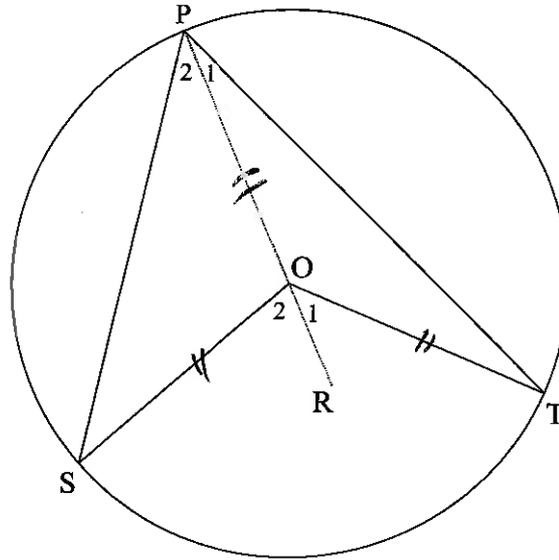
$$= 2x^2 + 4x(180 - x)$$

QUESTION/VRAAG 9

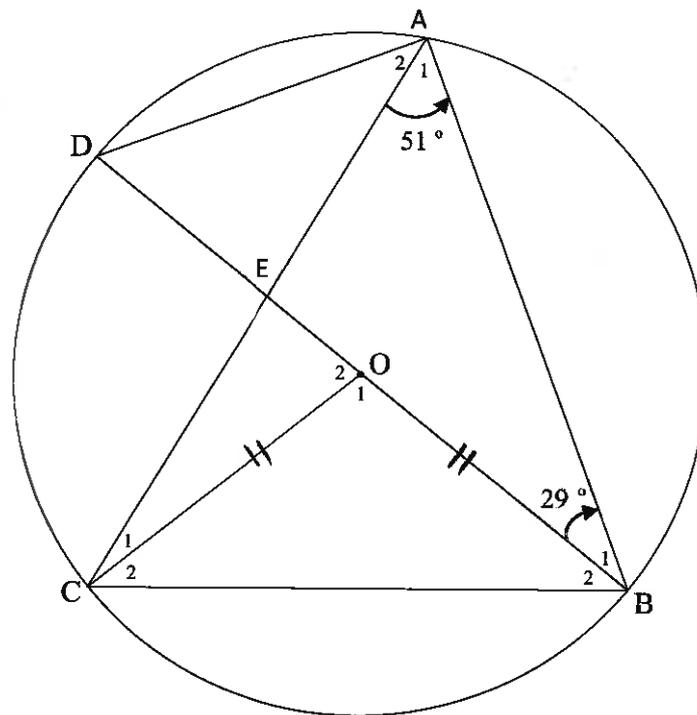


9.1.1	$\hat{Q} = 180^\circ - 145^\circ$ [opp \angle 's of cyclic quad] $= 35^\circ$	\checkmark S \checkmark R (2)
9.1.2	$\hat{R} = 180^\circ - 145^\circ$ [opp \angle 's of cyclic quad] or [\angle 's in the same segment] $= 35^\circ$	\checkmark S \checkmark R (2)
9.1.3	$\hat{P}_2 = \hat{R} = 35^\circ$ [alt \angle 's, PM \parallel SR]	\checkmark S / R (1)
9.2	$\therefore \hat{P}_2 = \hat{Q}$ \therefore PR is a tangent to circle PMQ [<u>converse tan-chord theorem</u>]	\checkmark R (1)
		[6]

QUESTION/VRAAG 10

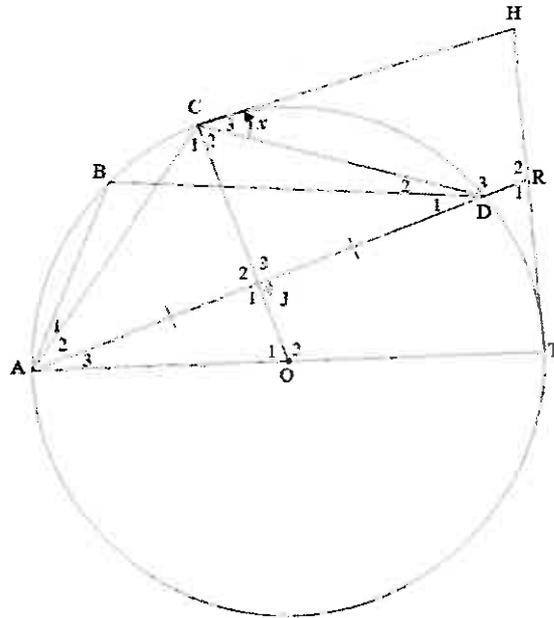


<p>10.1</p>	<p>Construction/Konstruksie: Draw/Trek POR Let $\hat{P}_2 = x$ $\hat{P}_2 = \hat{S}$ [angles opp = sides, radii] $\hat{O}_2 = 2x$ [exterior \angle of Δ] Let $\hat{P}_1 = y$ $\hat{P}_1 = \hat{T}$ [angles opp = sides] $\hat{O}_1 = 2y$ [exterior \angle of Δ] $\hat{O}_1 + \hat{O}_2 = 2x + 2y = 2(x + y)$ $\hat{P}_1 + \hat{P}_2 = x + y$ $\therefore \hat{SOT} = 2 \times \hat{SPT}$</p>	<p>✓ Construction/ Konstruksie ✓ S/R ✓ S/R ✓ S ✓ S (5)</p>
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10.2.1	$\hat{O}_1 = 102^\circ$ [angle at centre = 2 times angle at circumference / middelpuntshoek = 2 keer omtrekshoek]	✓ S ✓ R (2)
10.2.2	$\hat{A}_1 + \hat{A}_2 = 90^\circ$ $\hat{\text{in semi}} \hat{O} = 90^\circ$ $\hat{A}_2 = 39^\circ$	✓ S ✓ R (2)
10.2.3.	$\hat{D} = 61^\circ$ [sum of int \angle^s of Δ]	✓ S (1)
10.2.4	$\hat{ACB} = \hat{D} = 61^\circ$ [\angle^s in the same segment] $\hat{C}_2 = \frac{180^\circ - 102^\circ}{2}$ [sum of int \angle^s of Δ] $= 39^\circ$ [als opp = sides, radius] $\hat{ACO} = 61^\circ - 39^\circ$ $= 22^\circ$	✓ S/R ✓ S ✓ S (3)
		[13]

QUESTION/VRAAG 11



11.1	$\hat{C}_3 = \hat{A}_2 = x$ [tan - chord]	✓ S ✓ R (2)
11.2	$\hat{J}_3 = 90^\circ$ [line from centre to midpt of chord] $O\hat{C}H = 90^\circ$ [tan \perp rad] $CH \parallel JR$ [co - interior $\angle = 180^\circ$] $\therefore CHRJ$ is a trapezium [one propp sides \parallel] <p style="text-align: center;">OR/OF</p> $\hat{J}_4 = 90^\circ$ [line from centre to midpt of chord] $O\hat{C}H = 90^\circ$ [tan \perp rad] $CH \parallel JR$ [corresp $\angle =$] $\therefore CHRJ$ is a trapezium [one propp sides \parallel]	✓ S ✓ R ✓ S ✓ R ✓ S <p style="text-align: center;">OR/OF</p> ✓ S ✓ R ✓ S ✓ R ✓ S (5)
11.3	In $\triangle CJA$ and $\triangle CJD$ $\hat{J}_2 = \hat{J}_3$ [line from centre to midpt of chord] $AJ = JD$ [given] $CJ = CJ$ [common side] $\therefore \triangle CJA \cong \triangle CJD$ [SAS] $\hat{C}_1 = \hat{C}_2$ [$\triangle CJA \cong \triangle CJD$] OC bisects $A\hat{C}D$	✓ S ✓ S ✓ S (3)

<p>11.4</p>	<p>$\hat{C}_2 = 90^\circ - x$ [tan \perp rad] $\hat{B} = \hat{C}_1 + \hat{C}_2$ [\angle^s in the same segment] $= 90^\circ - x + 90^\circ - x$ [$\hat{C}_1 = \hat{C}_2$] $= 180^\circ - 2x$</p> <p>OR/OF</p> <p>$\hat{ADC} = x$ [alt $\angle =$, CH \parallel JR] $\hat{C}_1 + \hat{C}_2 = 180^\circ - 2x$ [sum of int \angle^s of Δ] $\hat{ABD} = 180^\circ - 2x$ [\angle^s in the same segment]</p>	<p>✓ S ✓ S (2)</p> <p>✓ S ✓ S (2)</p>
<p>11.5</p>	<p>$\hat{T} = 90^\circ$ [tan \perp rad] $\hat{CAO} = 90^\circ - x$ [\angle^s opp = sides] $x + \hat{A}_3 = 90^\circ - x$ $\hat{A}_3 = 90^\circ - 2x$</p> <p>$\therefore \hat{R}_2 = 90^\circ + 90^\circ - 2x$ [ext \angle^s of Δ] $= 180^\circ - 2x$</p>	<p>✓ S ✓ R ✓ S/R ✓ S ✓ S ✓ S (6) [18]</p>

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